DERIVATIVES ADDITIONAL QUESTIONS

CLASS 2

Question 1:

Suppose that you enter into a two-year forward contract on a non-dividend-paying stock when the stock price is INR 400 and the risk-free rate (annually compounded) is 10% per year. What do you expect the forward price to be?

(Source: FOD)

ANSWER:

The forward price is INR 400 multiplied by 1.10^2, which equals INR 484.

Question 2:

The cash price of a bond is INR 900. It is expected to provide a coupon of INR 30 in six months and 12 months. The risk-free rate for all maturities is 5% per year (with annual compounding). What is the 15-month forward price of the bond?

(Source: FOD)

ANSWER:

The present value of the income (INR) is 30 / ($1.05^{0.5}$) + 30 / (1.05^{1}) = 29.28 + 28.57 = 57.85 INR

The forward price of the bond is (900 - 55.78) x $1.05^{1.25}$ = 895.11 INR.

Question 3:

A stock index is 30,000, the risk-free rate is 8% per year, and the dividend yield on the index is 3% per year (both expressed with continuous compounding). What should the one-year futures price of the index be?

(Source: FOD)

ANSWER:

The one-year futures price with continuous compounding is calculated as follows:

Futures Price = Spot Price x e^((Risk-free Rate - Dividend Yield) x Time)

Futures Price = 30,000 x e^((0.08 - 0.03) x 1)

Futures Price ≈ 31,538.13

Answer the Following Questions from 4-6

Question 4:

A one-year forward contract to buy a non-dividend-paying stock is entered into when the stock price is INR 500, and the risk-free rate is 5% per year (with annual compounding). What is the forward price?

(Source: FOD)

ANSWER:

The forward price is 500 x 1.05 = 525 INR. We will call this K

Question 5:

Six months after the forward contract was entered into, the spot price is INR 560, and the risk-free rate is still 5% per year. What is the

a. New forward price, and

b. Gain/Loss on squaring off the forward contract?

(Source: FOD)

ANSWER:

a. The new forward price is 560 x $(1.05)^{0.5}$ = 573.83 INR. We will call this new F

b. The gain on squaring off forward is calculated as:

PV of difference between new F & K

(573.83 - 525) / $(1.05^{0.5})$ = 47.65 INR

Question 6:

If the above two questions related to futures contract (Instead of forward), calculate the gain on squaring off futures?

(Source: FOD)

ANSWER:

The gain of the forward contract is calculated as:

Difference between new F & K

(573.83 - 525) = 48.83 INR

Question 7:

Mr. Y, a seasoned investor, anticipates purchasing a portfolio of stocks in 120 days. Concerned about potential price increases during this period, he decides to hedge against this risk by entering into a 120-day forward contract on the XYZ Index. The current index level is 2,800. Given a continuously compounded dividend yield of 2.5% and a risk-free interest rate of 3.75%, answer the following questions:

i. Calculate the justified forward price on this contract.

ii. Suppose after 28 days of the purchase of the contract, the index value stands at 2,920. Determine the gain/loss on the above long position.

iii. If at the expiration of 120 days, the Index Value is 2,950, what will be the gain on the long position?

(Source: FOD)

ANSWER:

i. To calculate the justified forward price for the 120-day contract:

- Current Index Level (S0) = 2,800
- Risk-free Interest Rate (r) = 3.75%
- Continuously Compounded Dividend Yield (y) = 2.5%
- Time to Maturity (T) = 120/365 (since there are 365 days in a year)

Justified Forward Price (F0(T))

= S0 × exp^((r - y) × T)

= 2,800 × exp^((0.0375 - 0.025) × (120/365))

= 2,811.53

ii. After 28 days:

- Index Value (St) = 2,920
- Time to Maturity (T - t) = 92/365

New Forward Price (Ft(T))

= St × exp^((r - y) × (T - t))

= 2,920 × exp^((0.0375 - 0.025) × (92/365))

= 2,929.21

To calculate the gain on the long position:

Gain on Long Position = [Ft(T) - F0(T)] / exp^(r × (T - t))

= [2,929.21 - 2,811.53] / exp^(0.0375 × (92/365))

= 117.68/exp^(0.00945)

= 117.68/1.00950 = 116.58

So, the gain on the long position after 28 days is approximately INR 116.58

Note: *however ICAI has provided a different solution which is conceptually wrong because they have done the present value using (r-y) rather than r. So if we follow their solution, answer would be:*

Gain on Long position = Present value of the difference between Ft(T) and k
= [Ft(T) – k]/exp^(r-y)(T-t)
= [(2,929.21 - 2,811.53)/ exp^((0.0375 - 0.025) × (92/365)]
= 117.31

iii. At the expiration of 120 days:

Since the forward contract has reached maturity, the gain on the long position will be based on the final index value:

Gain on Long Position at Expiration

= Final Index Value (S_T) - K

= 2,950 - 2,811.53 ≈ 138.47

The gain on the long position at the contract's expiration is approximately 138.47.

Question 8:

The risk-free rate of interest is 7% per annum with continuous compounding, and the dividend yield on a stock index is 3.2% per annum. The current value of the index is 150. What is the 6-month futures price?

(Source: FOD)

ANSWER:

To calculate the 6-month futures price of the stock index, you can use the cost-of-carry model, which takes into account the risk-free interest rate and the dividend yield. The formula for the futures price (F) is as follows:

$$F = S \times e^{(r - y) \times T}$$

Where:

- F is the futures price.
- S is the current value of the stock index (150 in this case).
- r is the risk-free interest rate (7% per annum with continuous compounding).
- y is the dividend yield (3.2% per annum).
- T is the time to maturity (6 months, which is 0.5 years).

Now, let's calculate the futures price:

$F = 150 \times e^{(0.07 - 0.032) \times 0.5}$

$F = 150 \times e^{(0.038 \times 0.5)}$

$F = 150 \times e^{0.019}$

$F \approx 150 \times 1.0191$

$F \approx 152.87$

So, the 6-month futures price of the stock index is approximately 152.87.

Question 9:

Assume that the risk-free interest rate is 9% per annum with continuous compounding and that the dividend yield on a stock index varies throughout the year. In February, May, August, and November, dividends are paid at a rate of 5% per annum. In other months, dividends are paid at a rate of 2% per annum. Suppose that the value of the index on July 31 is 1,300. What is the futures price for a contract deliverable in December 31 of the same year?

Note: *Do calculation in months (not days)*

(Source: FOD)

ANSWER:

To calculate the futures price for a contract deliverable on December 31 of the same year, taking into account the varying dividend yields throughout the year, you can use the cost-of-carry model.

This model considers the risk-free interest rate and the expected dividends during the period. Here's how to calculate it:

1. **First,** determine the time to maturity (T) in years. The contract is deliverable on December 31, and the current date is July 31. Therefore, T is 5/12 (since there are 5 months remaining until December 31).

2. **Next,** calculate the expected dividends during the period from July 31 to December 31:

Month	Annualised dividend yield
August	5%
September	2%
October	2%
November	5%
December	2%

$y = (5 + 2 + 2 + 5 + 2)/5 = 3.2\%$

Now, use the cost-of-carry model to calculate the futures price (F):

$F = S \times e^{((r - y) \times T)} - D$

Where:

- F is the futures price.
- S is the current value of the stock index (1300).
- r is the risk-free interest rate (9% per annum with continuous compounding).
- y is the expected dividend yield= 3.2%.
- T is the time to maturity (5/12 years).

Now, calculate F:

$F = 1300 \times e^{((0.09 - 0.032) \times (5/12))}$

$F = 1300 \times e^{(0.024167)}$

$= 1331.80$

So, the futures price for a contract deliverable on December 31 of the same year is approximately 1331.80.